ON MHD FLOW OF A RAREFIED GAS NEAR AN ACCELERATED PLATE

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ABSTRACT. An analysis of mhd flow of an electrically conducting, incompressible, viscous rarefied gas past an accelerated plate is carried out when the magnetic field is fixed relative to the (a) fluid (case I) (b) plate (case II). Expressions for velocity profiles and the drag are derived in closed form. It is observed that there is a decrease in velocity when there is an increase in the strength of the magnetic field and the rarefraction parameter h_1. An increase in h_1 leads to a decrease in the skin-friction. An increase in the strength of the magnetic field leads to an increase in the skin-friction in case (I) whereas in case (II), there is a dec easerin skin-friction.

INTRODUCTION

Rossow (1958) discussed MHD Rayleigh's problem wherein an induced magnetic field was neglected. This was generalized by Gupta (1960), Soundalgekar (1965), Pop (1968) to the case of the MHD flow past an accelerated plate. In all these problems, the flow of the normal density fluids was considered. In the present age of high altitude flights, the study of rarefied gases is receiving attention of a number of researchers. In case of the slightly rarefied gases, the physical aspect of the problem can be analysed by solving the Navier-Stokes equations under the first order velocity slip boundary conditions at the boundaries. Such an hydrodynamic attempt was made by Schaaf (1950) for Rayleigh's problem under first order velocity slip boundary conditions. The corresponding MHD aspect of this problem was recently discussed by the present authors (1969).

The object of this paper is to study the flow of an electrically conducting rarefied gas past an accelerated plate under transverse magnetic field. In the next section the problem is solved for velocity field in the case when the magnetic field lines of force are fixed relative to the fluid and relative to the plate. The velocity profiles are shown on graphs and the numerical values of the skin-friction are entered in tables. Lastly, the conclusions are set out.

MATHEMATICAL ANALYSIS

An infinite plate is assumed to be accelerated in the x-direction. The y-coordinate is taken perpendicular to it. The magnetic field is assumed to be

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applied parallel to the y-axis. The fluid and the plate are assumed to be stationary at $t < 0$ and the plate starts moving at $t = 0$. Then neglecting the induced fields, the governing momentum equation for the magnetic lines of force fixed relative to the fluid are (Gupta, 1960).

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \quad \ldots (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad \ldots (2)$$

where B_0 is the magnetic field and σ is the electrical conductivity of the fluid.

The boundary conditions for the present problem are

$$u = o \text{ everywhere for } t < 0 \quad \ldots (3)$$

$$\left.\begin{aligned} & u = At^n + \xi_u \frac{\partial u}{\partial y} \quad \text{at } y = 0 \\ & u \to 0 \text{ as } \quad y \to \infty \end{aligned}\right\} \quad \ldots (3a)$$

If Laplace-transform of u is defined as

$$\bar{u} = \int_0^\infty u(y, t) e^{-pt} dt \qquad (p > 0)$$

then the Laplace transform of (1) and (3a) with respect to t is as follows.

$$\frac{d^2\bar{u}}{dy^2} - \left(\frac{p+m}{\nu}\right)\bar{u} = 0, \quad \ldots (4)$$

where $m - (\sigma B_0^2)/\rho$

and

$$\left.\begin{aligned} & \bar{u} = \frac{n\,!\,A}{p^{n+1}} + \xi_u \frac{d\bar{u}}{dy} \\ & \bar{u} \to 0 \quad \text{as} \quad y \to \infty. \end{aligned}\right\} \quad \ldots (5)$$

For uniformly accelerated plate, ($n = 1$), the solution of (4) subject to the boundary condition (5) in non-dimensional form on taking inverse, is

$$\phi(\eta, a_0) = \frac{u(y, t)}{At} = \left\{\frac{(2a_0+\eta)(1-h_1a_0)+h_1}{4a_0(1-h_1a_0)^2}\right\} e^{\eta a_0} . \operatorname{erfc}\left(\frac{\eta}{2} + a_0\right)$$

$$+ \left\{\frac{(2a_0-\eta)(1+h_1a_0)-h_1}{4a_0(1+h_1a_0)^2}\right\} e^{-\eta a_0} . \operatorname{erfc}\left(\frac{\eta}{2} - a_0\right)$$

$$-\frac{h_1 e^{-a^2_0}}{(1-h_1^2a_0^2)}\left[\frac{e^{-\eta^2/4}}{\sqrt{\pi}} + \frac{h_1\, e^{\left(\frac{\eta}{h_1} + \frac{1}{h_1^2}\right)} . \operatorname{erfc}\left(\frac{\eta}{2} + \frac{1}{h_1}\right)}{(1-h_1^2a_0^2)}\right], \quad \ldots (6)$$

where $$a_0 = \sqrt{mt}\,, \quad \eta = \frac{y}{\sqrt{\nu t}}\,, \quad h_1 = \frac{\xi_u}{\sqrt{\nu t}}$$

and ξ_u has its meaning as defined in Soundalgekar *et al* (1968). When $h_1 \to 0$, (6) reduces to Gupta's (1960) case. The skinfriction in non-dimensional form is given by

$$\tau = -\left(\frac{\partial\phi}{\partial\eta}\right)_{\eta=0}, \qquad (7)$$

where $$\tau = \frac{\tau^*}{\mu A}\sqrt{\frac{\nu}{t}}\,.$$

Hence from (6) and (7), we have

$$\tau = \left\{\frac{2a_0^2(1+h_1a_0)+1}{4a_0(1+h_1a_0)^2}\right\}\operatorname{erfc}(-a_0) + \left\{\frac{h_1^2(1-a_0^2)+1}{\sqrt{\pi}(1-h_1^2a_0^2)^2}\right\}e^{-a_0^2} +$$
$$+ \frac{h_1^2e^{-a_0^2}}{(1-h_1^2a_0^2)}\left\{\frac{1}{h_1}\,e^{\left(\frac{1}{h_1^2}\right)}\,.\operatorname{erfc}\left(\frac{1}{h_1}\right) - \frac{1}{\sqrt{\pi}}\right\} -$$
$$-\left\{\frac{2a_0^2(1-h_1a_0)+1}{4a_0(1-h_1a_0)^2}\right\}\operatorname{erfc}(a_0). \qquad (8)$$

The velocity profiles from (6) are shown in figure 1 and the numerical values of τ from (8) are entered in table 1.

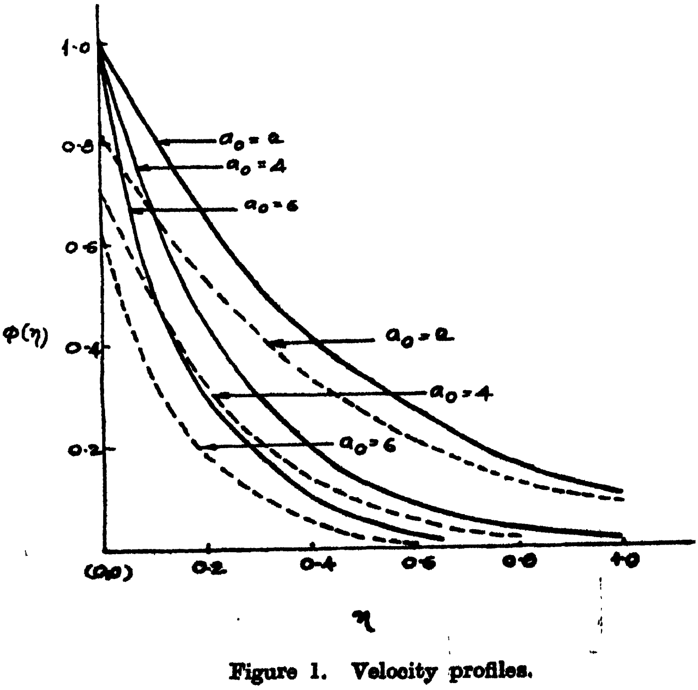

Figure 1. Velocity profiles.

Table 1

Values of $-\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0}$

h_1 \ a_0	2	4	6
0.0	2.2498	4.1250	6.0833
0.1	1.8390	2.9196	3.7840
0.2	1.5023	2.2588	2.7450

We now consider the case of the magnetic lines of force fixed relative to the plate.

The momentum equation in this case (Soundalgekar, 1965) is

$$\frac{\partial u}{\partial t} + m(u - At^n) = \nu \frac{\partial^2 u}{\partial y^2} \qquad .. \quad (9)$$

Proceeding as in the above case, the solution in non-dimensional form for uniformly accelerated plate is given by

$$\phi(\eta, a_0) = \frac{u(y, t)}{At} =$$

$$= 1 + \frac{1}{2a_0^2}\left\{2e^{-a_0^2}\left[\operatorname{erfc}\left(\frac{\eta}{2}\right) - \frac{e^{\left(\frac{\eta}{h_1} + \frac{1}{h_1^2}\right)} . \operatorname{erfc}\left(\frac{\eta}{h_1} + \frac{1}{h_1}\right)}{(1 - h_1^2 a_0^2)}\right]\right.$$

$$\left. + \frac{e^{-\eta a_0} . \operatorname{erfc}\left(\frac{\eta}{2} - a_0\right)}{1 + h_1 a_0} + \frac{e^{\eta a_0} . \operatorname{erfc}\left(\frac{\eta}{2} + a_0\right)}{1 - h_1 a_0} - 2\right\} \qquad .. \quad (10)$$

The skin friction in this case is now given by

$$\tau = -\frac{1}{a_0^2}\left[\left\{\frac{1}{\sqrt{\pi}} - \frac{e^{1/h_1^2} . \operatorname{erfc}\left(\frac{1}{h_1}\right)}{h_1(1 - h_1^2 a_0^2)}\right\} e^{-a_0^2} + \right.$$

$$\left. + \frac{a_0}{(1 - h_1^2 a_0^2)}\left\{h_1 a_0 - \operatorname{erf}(a_0)\right\}\right] \qquad .. \quad (11)$$

when $h_1 \to 0$, (10) and (11) reduce to the case considered in Soundalgekar (1965).

The velocity profiles from (10) are shown in figure 2 and the numerical values of $-\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0}$ are entered in table 2.

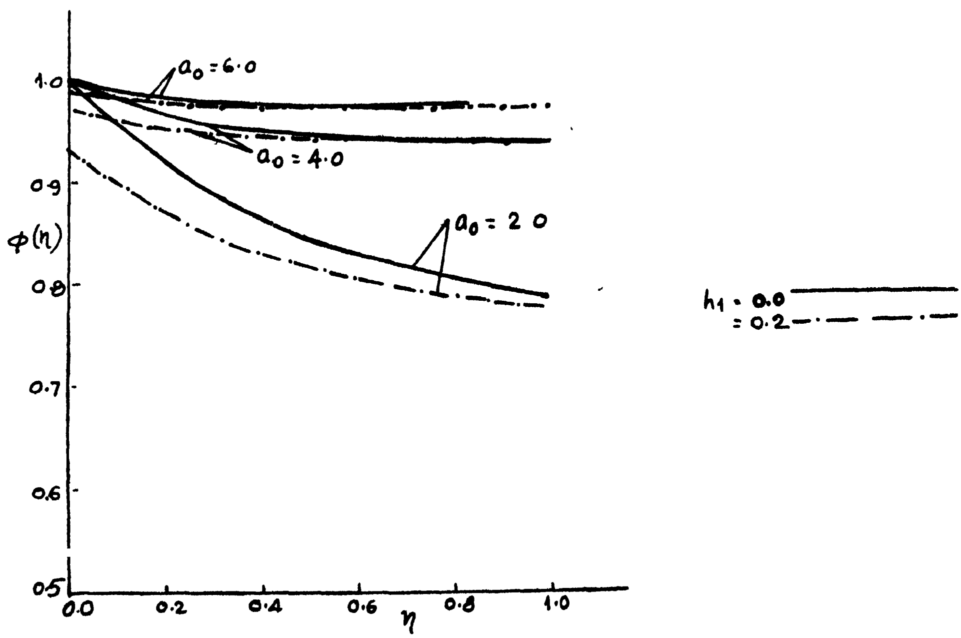

Figure 2. Velocity profiles.

Table 2

Value of $-\left(\frac{\partial\phi}{\partial\eta}\right)_{\eta=0}$

h_1 \ a_0	2	4	6
0.0	0.4951	0.2500	0.1666
0.2	0.3548	0.0694	0.0253
0.3	0.3106	0.0568	0.0099

CONCLUSIONS

1) In both the cases, the velocity decreases with increasing the magnetic field strength and also with increasing the rarefaction parameter h_1. But in case of the magnetic field fixed relative to the plate, the velocity profiles become almost parallel to the moving plate at large values of a_0, the magnetic field parameter.

2) In the first case (table 1), an increase in a_0 leads to an increase in the skin-friction whereas an increase in h_1 leads to a decrease in the skin-friction.

3) In the second case (table 2), the effect of h_1 being the same, there is a decrease in the skin-friction with increasing a_0.

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